

CBSE Class 11 Mathematics
Important Questions
Chapter 3
Trigonometric Functions

1 Marks Questions

1. Find the radian measure corresponding to $5^\circ 37' 30''$

Ans. $\left(\frac{\pi}{32}\right)^c$

2. Find the degree measure corresponding to $\left(\frac{11}{16}\right)^c$

Ans. $39^\circ 22' 30''$

3. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°

Ans. $\frac{5\pi}{12}$ cm

4. Find the value of $\frac{19\pi}{3}$

Ans. $\sqrt{3}$

5. Find the value of $\sin(-1125^\circ)$

Ans. $\frac{-1}{\sqrt{2}}$

6. Find the value of $\tan 15^\circ$

Ans. $2 - \sqrt{3}$

7. If $\sin A = \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$, find $\cos A$

Ans. $-\frac{4}{5}$

8. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$ then find the value of $A + B$.

Ans. 45°

9. Express $\sin 12\theta + \sin 4\theta$ as the product of sines and cosines.

Ans. $2 \sin 8\theta \cos 4\theta$

10. Express $2 \cos 4x \sin 2x$ as an algebraic sum of sines or cosines.

Ans. $\sin 6x - \sin 2x$

11. Write the range of $\cos \theta$

Ans. $[-1, 1]$

12. What is domain of $\sec \theta$?

Ans. $\mathbb{R} - \left\{ (2n+1)\frac{\pi}{2}; n \in \mathbb{Z} \right\}$

13. Find the principal solutions of $\cot x = 3$

Ans. $\frac{5\pi}{6}, \frac{11\pi}{6}$

14. Write the general solution of $\cos \theta = 0$

Ans. $(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$

15. If $\sin x = \frac{\sqrt{5}}{3}$ and $0 < x < \frac{\pi}{2}$ find the value of $\cos 2x$

Ans. $-\frac{1}{9}$

16. If $\cos x = \frac{-1}{3}$ and x lies in quadrant III, find the value of $\sin \frac{x}{2}$

Ans. $\frac{\sqrt{6}}{3}$

17. Convert into radian measures. $-47^\circ 30'$

Ans. $-47^\circ 30' = -\left(47 + \frac{30}{60}\right)^\circ$

$= -\left(47\frac{1}{2}\right)^\circ$

$= -\left(\frac{95}{2} \times \frac{\pi}{180}\right) \text{ radians}$

$= -\frac{19\pi}{72} \text{ radians.}$

18. Evaluate $\tan 75^\circ$.

Ans. $\tan 75 = \tan (45 + 30)$

$$\begin{aligned} &= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} \\ &= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \end{aligned}$$

19. Prove that $\sin (40 + \theta) \cdot \cos (10 + \theta) - \cos (40 + \theta) \cdot \sin (10 + \theta) = \frac{1}{2}$

Ans. L. H. S = $\sin (40 + \theta) \cdot \cos (10 + \theta) - \cos (40 + \theta) \cdot \sin (10 + \theta)$

$$= \sin [40 + \cancel{\theta} - 10 - \cancel{\theta}] = \sin 30 = \frac{1}{2}$$

20. Find the principal solution of the eq. $\sin x = \frac{\sqrt{3}}{2}$

Ans. $\sin x = \frac{\sqrt{3}}{2}$

$$\sin x = \sin \frac{\pi}{3}$$

$$x = \frac{\pi}{3}$$

$$\sin x = \sin \left(\pi - \frac{\pi}{3} \right)$$

$$x = \frac{2\pi}{3}$$

21. Prove that $\cos \left(\frac{\pi}{4} + x \right) + \cos \left(\frac{\pi}{4} - x \right) = \sqrt{2} \cos x$

Ans. L. H. S = $\cos \left(\frac{\pi}{4} + x \right) + \cos \left(\frac{\pi}{4} - x \right)$

$$2 \cos \frac{\pi}{4} \cdot \cos x \quad [\because \cos (A + B) + \cos (A - B) = 2 \cos A \cdot \cos B]$$

$$= 2 \cdot \frac{1}{\sqrt{2}} \cdot \cos x = \sqrt{2} \cos x$$

22. Convert into radian measures. $-37^\circ 30'$

Ans. $-37^\circ 30' = - \left(37 + \frac{30'}{60} \right)^\circ$

$$= - \left(\frac{75}{2} \right)^\circ$$

$$= - \frac{75}{2} \times \frac{\pi}{180} \text{ radian}$$

$$= - \frac{5\pi}{24}$$

23. Prove $\sin (n+1) x \sin (n+2) x + \cos (n+1) x \cdot \cos (n+2) x = \cos x$

Ans. L.H.S = $\cos (n+1) x \cos (n+2) x + \sin (n+1) x \sin (n+2) x$

$$= \cos [(n+1)x - (n+2)x]$$

$$= \cos x$$

24. Find the value of $\sin \frac{31\pi}{3}$

Ans. $\sin \frac{31\pi}{3} = \sin \left(10\pi + \frac{\pi}{3} \right)$

$$= \sin \left(2\pi \times 5 + \frac{\pi}{3} \right)$$

$$= \sin \frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2}$$

25. Find the principal solution of the eq. $\tan x = \frac{-1}{\sqrt{3}}$

Ans. $\tan x = -\frac{1}{\sqrt{3}}$

$$\tan x = \tan \left(\pi - \frac{\pi}{6} \right)$$

$$x = \frac{5\pi}{6}$$

$$\tan x = -\frac{1}{\sqrt{3}}$$

$$\tan x = \tan \left(2\pi - \frac{\pi}{6} \right)$$

$$x = \frac{11\pi}{6}$$

26. Convert into radian measures. $5^{\circ} 37' 30''$

$$\text{Ans. } 5^0 37^1 30^{11} = 5^0 + \left(37 + \frac{30}{60}\right)^1$$

$$= 5^0 + \left(\frac{75}{2}\right)^1$$

$$= 5^0 + \left(\frac{75}{2} \times \frac{1}{60}\right)^0$$

$$= 5^0 + \left(\frac{5}{8}\right)^0$$

$$= \left(5\frac{5}{8}\right)^0$$

$$= \left(\frac{45}{8}\right)^0$$

$$= \frac{45}{8} \times \frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{32} \text{ radian.}$$

$$27. \text{ Prove } \cos 70^\circ \cdot \cos 10^\circ + \sin 70^\circ \cdot \sin 10^\circ = \frac{1}{2}$$

$$\text{Ans. L. H. S.} = \cos (70 - 10) = \cos 60 = \frac{1}{2}$$

$$28. \text{ Evaluate } 2 \sin \frac{\pi}{12}.$$

$$\text{Ans. } 2 \sin \frac{\pi}{12} = 2 \sin \left[\frac{\pi}{4} - \frac{\pi}{6} \right]$$

$$= 2 \left[\sin \frac{\pi}{4} \cos \frac{\pi}{6} - \cos \frac{\pi}{4} \sin \frac{\pi}{6} \right]$$

$$= 2 \left[\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} \right]$$

$$= \frac{\sqrt{3}-1}{\sqrt{2}}$$

29. Find the solution of $\sin x = -\frac{\sqrt{3}}{2}$

Ans. $\sin x = -\frac{\sqrt{3}}{2}$

$$\sin x = \sin \left(\pi + \frac{\pi}{3} \right)$$

$$\sin x = \sin \frac{4\pi}{3}$$

if $\sin \theta = \sin \alpha$

$$\theta = n\pi + (-1)^n \cdot \alpha$$

$$x = n\pi + (-1)^n \cdot \frac{4\pi}{3}$$

30. Prove that $\frac{\cos 9^\circ - \sin 9^\circ}{\cos 9^\circ + \sin 9^\circ} = \tan 36^\circ$

Ans. L. H. S = $\tan 36^\circ$

$$= \tan (45^\circ - 9^\circ)$$

$$= \frac{1 - \tan 9^\circ}{1 + \tan 9^\circ}$$

$$= \frac{\cos 9^\circ - \sin 9^\circ}{\cos 9^\circ + \sin 9^\circ}$$

31. Find the value of $\tan \frac{19\pi}{3}$.

Ans. $\tan \frac{19\pi}{3} = \tan \left(6\pi + \frac{\pi}{3} \right)$

$$= \tan \left(3 \times 2\pi + \frac{\pi}{3} \right)$$

$$= \tan \frac{\pi}{3} = \sqrt{3}$$

32. Prove $\cos 4x = 1 - 8 \sin^2 x \cdot \cos^2 x$

Ans. L. H. S = $\cos 4x$

$$= 1 - 2 \sin^2 2x \quad [\because \cos 2\theta = 1 - 2\sin^2 \theta]$$

$$= 1 - 2 (\sin 2x)^2$$

$$= 1 - 2 (2 \sin x \cdot \cos x)^2$$

$$= 1 - 2 (4 \sin^2 x \cdot \cos^2 x)$$

$$= 1 - 8 \sin^2 x \cdot \cos^2 x.$$

33. Prove

$$\frac{\cos(\pi+x) \cdot \cos(-x)}{\sin(\pi-x) \cdot \cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$$

$$\text{Ans.L. H. S} = \frac{-\cos x \cdot \cos x}{-\sin x \cdot \sin x} = \cot^2 x.$$

34. Prove that $\tan 56^\circ = \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$

$$\text{Ans.L. H. S} = \tan 56^\circ$$

$$= \tan(45^\circ + 11^\circ)$$

$$= \frac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \cdot \tan 11^\circ}$$

$$= \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ}$$

$$= \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \text{R H S}$$

35. Prove that $\cos 105^\circ + \cos 15^\circ = \sin 75^\circ - \sin 15^\circ$

$$\text{Ans.L. H. S} = \cos 105^\circ + \cos 15^\circ$$

$$= \cos(90^\circ + 15^\circ) + \cos(90^\circ - 75^\circ)$$

$$= -\sin 15^\circ + \sin 75^\circ$$

$$= \sin 75^\circ - \sin 15^\circ$$

36. Find the value of $\cos(-1710^\circ)$.

$$\text{Ans. } \cos(-1710^\circ) = \cos(1800-90)[\cos(-\theta) = \cos \theta]$$

$$= \cos [5 \times 360 + 90]$$

$$= \cos \frac{\pi}{2} = 0$$

37. A wheel makes 360 revolutions in 1 minute. Through how many radians does it turn in 1 second.

Ans. N. of revolutions made in 60 sec. = 360

$$\text{N. of revolutions made in 1 sec} = \frac{360}{60} = 6$$

$$\text{Angle moved in 6 revolutions} = 2\pi \times 6 = 12\pi$$

38. Prove $\sin^2 6x - \sin^2 4x = \sin 2x \cdot \sin 10x$.

$$\text{Ans. L. H. S} = \sin^2 6x - \sin^2 4x$$

$$= \sin (6x + 4x) \cdot \sin (6x - 4x)$$

$$= \sin 10x \cdot \sin 2x$$

$$\text{39. Prove that } \frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \cdot \tan 66^\circ} = -1$$

$$\text{Ans. L. H. S} = \tan (69^\circ + 66^\circ)$$

$$= \tan (135^\circ)$$

$$= \tan (90^\circ + 45^\circ)$$

$$= -\tan 45^\circ$$

$$= -1$$

40. Prove that $\frac{\sin x}{1 + \cos x} = \tan \frac{x}{2}$

Ans. L. H. S

$$\frac{2\sin\frac{x}{2} \cdot \cos\frac{x}{2}}{2\cos^2\frac{x}{2}}$$
$$= \tan \frac{x}{2}$$



CBSE Class 12 Mathematics

Important Questions

Chapter 3

Trigonometric Functions

4 Marks Questions

Prove the following Identities

1. The minute hand of a watch is 1.5 cm long. How far does it tip move in 40 minute?

Ans. $r = 1.5$ cm

Angle made in 60 mint = 360^0

Angle made in 1 min = $\frac{360}{60} = 60^0$

Angle made in 40 mint = 6×40

= 240^0

$$\Theta = \frac{l}{r}$$

$$240 \times \frac{\pi}{3} = \frac{l}{1.5}$$

$$\frac{4 \times 3.14}{3} = \frac{l}{1.5}$$

$$2 \times 3.14 = l$$

$$6.28 = l$$

$$l = 6.28 \text{ cm}$$

2. Show that $\tan 3x - \tan 2x - \tan x = \tan 3x \cdot \tan 2x \cdot \tan x$

Ans. Let $3x = 2x + x$

$$\tan 3x = \tan (2x + x)$$

$$\frac{\tan 3x}{1} = \frac{\tan 2x + \tan x}{1 - \tan 2x \cdot \tan x}$$

$$\tan 3x (1 - \tan 2x \cdot \tan x) = \tan 2x + \tan x$$

$$\tan 3x - \tan 3x \cdot \tan 2x \cdot \tan x = \tan 2x + \tan x$$

$$\tan 3x \cdot \tan 2x \cdot \tan x = \tan 3x - \tan 2x - \tan x$$

3. Find the value of $\tan \frac{\pi}{8}$.

Ans. Let $x = \frac{\pi}{8}$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\tan \left(2 \cdot \frac{\pi}{8} \right) = \frac{2 \tan \pi/8}{1 - \tan^2 \pi/8}$$

$$1 = \frac{2 \tan \pi/8}{1 - \tan^2 \pi/8}$$

$$\text{put } \tan \pi/8 = t$$

$$\frac{1}{1} = \frac{2t}{1 - t^2}$$

$$2t = 1 - t^2$$

$$t^2 + 2t - 1 = 0$$

$$t = \frac{-2 \pm 2\sqrt{2}}{2 \times 1}$$

$$= -1 \pm \sqrt{2}$$

$$= \pm \sqrt{2} - 1$$

$$= \sqrt{2} - 1 \text{ or } -\sqrt{2} - 1$$

$$\tan \pi/8 = \sqrt{2} - 1$$

4. Prove that $\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x + \tan y}{\tan x - \tan y}$

Ans. L.H.S = $\frac{\sin(x+y)}{\sin(x-y)}$

$$= \frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y}$$

Dividing N and D by $\cos x \cos y$

$$= \frac{\tan x + \tan y}{\tan x - \tan y}$$

5. If in two circles, arcs of the same length subtend angles 60° and 75° at the centre find the ratio of their radii.

Ans. $\theta = \frac{l}{r_1}$

$$60 \times \frac{\pi}{180} = \frac{l}{r_1}$$

$$r_1 = \frac{3l}{\pi} \quad (1)$$

$$\theta = \frac{l}{r_2}$$

$$75 \times \frac{\pi}{18} = \frac{l}{r_2}$$

$$r_2 = \frac{12l}{5\pi} \quad (2)$$

$$(1) \div (2)$$

$$\frac{r_1}{r_2} = \frac{\frac{3l}{\pi}}{\frac{12l}{5\pi}}$$

$$= \frac{3l}{\pi} \times \frac{5\pi}{12l}$$

$$= 5:4$$

6. Prove that $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

Ans. L.H.S. = $\cos 6x$

$$= \cos 2(3x) = 2 \cos^2 3x - 1 = \cos 2(3x)$$

$$= 2(4 \cos^3 x - 3 \cos x)^2 - 1$$

$$= 2[16 \cos^6 x + 9 \cos^2 x - 24 \cos^4 x] - 1$$

$$= 32 \cos^6 x + 18 \cos^2 x - 48 \cos^4 x - 1$$

$$= 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$$

7. Solve $\sin 2x + \sin 4x + \sin 6x = 0$

Ans. $\sin 6x + \sin 2x - \sin 4x = 0$

$$2 \sin \left(\frac{6x+2x}{2} \right) \cdot \cos \left(\frac{6x-2x}{2} \right) - \sin 4x = 0$$

$$2 \sin 4x \cdot \cos 2x - \sin 4x = 0$$

$$\sin 4x [2 \cos 2x - 1] = 0$$

$$\sin 4x = 0$$

$$4x = n\pi$$

$$x = \frac{n\pi}{4}$$

$$2 \cos 2x - 1 = 0$$

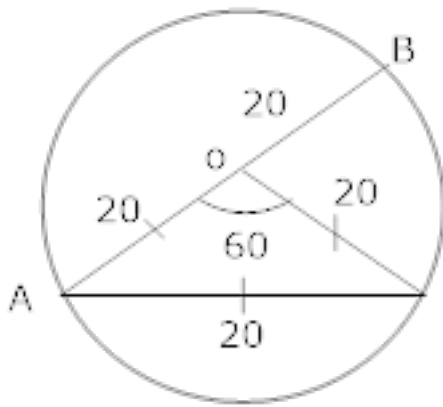
$$\cos 2x = \cos \frac{\pi}{3}$$

$$2x = 2n\pi \pm \frac{\pi}{3}$$

$$x = n\pi \pm \frac{\pi}{6}$$

8. In a circle of diameter 40cm, the length of a chord is 20cm. Find the length of minor arc of the chord.

Ans.



$$\theta = 60^\circ$$

$$\theta = \frac{l}{r}$$

$$60^\circ \times \frac{\pi}{180^\circ} = \frac{l}{20}$$

$$l = \frac{20\pi}{3} \text{ cm. s}$$

9. Prove that $\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

Ans. L. H. S = $\tan 4x$

$$= \frac{2 \tan 2x}{1 - \tan^2 2x}$$

$$= \frac{2 \cdot \frac{2 \tan x}{1 - \tan^2 x}}{1 - \left(\frac{2 \tan x}{1 - \tan^2 x} \right)^2}$$

$$\begin{aligned}
&= \frac{\frac{4 \tan x}{1 - \tan^2 x}}{\frac{(1 - \tan^2 x)^2 - 4 \tan^2 x}{(1 - \tan^2 x)^2}} \\
&= \frac{4 \tan x}{(1 - \tan^2 x)} \times \frac{(1 - \tan^2 x)}{1 + \tan^4 x - 2 \tan^2 x - 4 \tan^2 x} \\
&= \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}
\end{aligned}$$

10. Prove that $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \left(\frac{x+y}{2} \right)$

Ans. L. H. S = $(\cos x + \cos y)^2 + (\sin x - \sin y)^2$

$$\begin{aligned}
&= \left(2 \cos \frac{x+y}{2} \cdot \cos \frac{x-y}{2} \right)^2 + \left(2 \cos \left(\frac{x+y}{2} \right) \cdot \sin \left(\frac{x-y}{2} \right) \right)^2 \\
&= 4 \cos^2 \frac{x+y}{2} \cdot \cos^2 \left(\frac{x-y}{2} \right) + 4 \cos^2 \frac{x+y}{2} \cdot \sin^2 \frac{x-y}{2} \\
&= 4 \cos^2 \left(\frac{x+y}{2} \right) \left[\cos^2 \frac{x-y}{2} + \sin^2 \frac{x-y}{2} \right] \\
&= 4 \cos^2 \left(\frac{x+y}{2} \right)
\end{aligned}$$

11. If $\cot x = -\frac{5}{12}$, x lies in second quadrant find the values of other five trigonometric functions.

Ans. $\cot x = -\frac{5}{12}$

$$\tan x = -\frac{12}{5}$$

$$\sec^2 x = 1 + \tan^2 x$$

$$\sec x = \pm \frac{13}{5}$$

$$\sec x = -\frac{13}{5} \quad [\because x \text{ lies in IIInd quad.}]$$

$$\cos x = -\frac{5}{13}$$

$$\sin x = \tan x \cdot \cos x$$

$$= \frac{-12}{5} \times \left(\frac{-5}{13}\right) = \frac{12}{13}$$

$$\operatorname{Cosec} x = \frac{13}{12}$$

12. Prove that $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$

Ans. L. H. S = $\frac{\sin 5x + \sin x - 2\sin 3x}{\cos 5x - \cos x}$

$$= \frac{2\sin 3x \cdot \cos 2x - 2\sin 3x}{-2\sin 3x \cdot \sin 2x}$$

$$= \frac{\cancel{2\sin 3x} (\cos 2x - 1)}{-\cancel{2\sin 3x} \cdot \sin 2x}$$

$$= \frac{\cancel{2}(1 - \cos 2x)}{\cancel{2}\sin 2x}$$

$$= \frac{\cancel{2 \sin}^2 x}{\cancel{2 \sin} x \cdot \cos x}$$

$$= \frac{\sin x}{\cos x} = \tan x$$

13. Prove that $\sin x + \sin 3x + \sin 5x + \sin 7x = 4 \cos x \cdot \cos 2x \cdot \sin 4x$

Ans. L. H. S. = $\sin x + \sin 3x + \sin 5x + \sin 7x$

$$= \sin x + \sin 7x + \sin 3x + \sin 5x$$

$$= 2 \sin \left(\frac{x+7x}{2} \right) \cdot \cos \left(\frac{x-7x}{2} \right) + 2 \sin \left(\frac{3x+5x}{2} \right) \cos \left(\frac{3x-5x}{2} \right)$$

$$= 2 \sin 4x \cdot \cos 3x + 2 \sin 4x \cdot \cos x$$

$$= 2 \sin 4x [\cos 3x + \cos x]$$

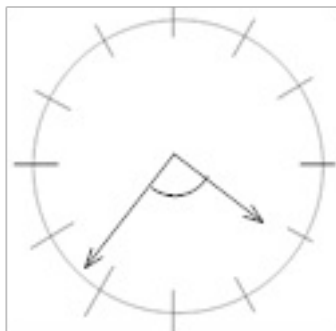
$$= 2 \sin 4x \left[2 \cos \left(\frac{3x+x}{2} \right) \cdot \cos \left(\frac{3x-x}{2} \right) \right]$$

$$= 2 \sin 4x [2 \cos 2x \cdot \cos x]$$

$$= 4 \cos x \cdot \cos 2x \cdot \sin 4x$$

14. Find the angle between the minute hand and hour hand of a clock when the time is 7.20.

Ans. Angle made by mint hand in 15 mint = $15 \times 6 = 90^\circ$



Angle made by hour hand in 1 hr = 30^0

$$\text{in 60 minute} = \frac{30}{60} = \frac{1}{2}$$

$[\because \text{Angle Traled by hr hand in 12 hr} = 360^0]$

$$\text{in 20 minute} = \frac{1}{2} \times 20 = 10^0$$

$$\text{Angle made} = 90 + 10 = 100^0$$

15.Show that $\sqrt{2 + \sqrt{2 + 2 \cos 4\theta}} = 2 \cos \theta$

$$\text{Ans.L H. S} = \sqrt{2 + \sqrt{2 + 2 \cos 4\theta}}$$

$$= \sqrt{2 + \sqrt{2(1 + \cos 4\theta)}}$$

$$= \sqrt{2 + \sqrt{2 \cdot 2 \cos^2 2\theta}}$$

$$= \sqrt{2 + 2 \cos 2\theta}$$

$$= \sqrt{2(1 + \cos 2\theta)}$$

$$= \sqrt{2 \cdot 2 \cos^2 \theta}$$

$$= 2 \cos \theta$$

16.Prove that $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

$$\text{Ans.L. H. S} = \cot 4x (\sin 5x + \sin 3x)$$

$$= \frac{\cos 4x}{\sin 4x} \left[2 \sin \frac{5x+3x}{2} \cdot \cos \frac{5x-3x}{2} \right]$$

$$= \frac{\cos 4x}{\cancel{\sin 4x}} 2 \cancel{\sin 4x} \cos x$$

$$= 2 \cos 4x \cdot \cos x$$

$$\text{R. H. S} = \cot x (\sin 5x - \sin 3x)$$

$$= \frac{\cos x}{\sin x} \left[2 \cos \frac{5x+3x}{2} \cdot \sin \frac{5x-3x}{2} \right]$$

$$= \frac{\cos x}{\cancel{\sin x}} \left[2 \cos 4x \cdot \cancel{\sin x} \right]$$

$$= 2 \cos 4x \cdot \cos x$$

$$\text{L. H. S} = \text{R. H. S}$$

CBSE Class 12 Mathematics

Important Questions

Chapter 3

Trigonometric Functions

6 Marks Questions

1. Find the general solution of $\sin 2x + \sin 4x + \sin 6x = 0$

Ans. $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$

2. Find the general solution of $\cos \theta \cos 2\theta \cos 3\theta = \frac{1}{4}$

Ans. $(2n+1)\frac{\pi}{8}, n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$

3. If $\sin \alpha + \sin \beta = a$ and $\cos \alpha + \cos \beta = b$ show that $\cos (\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$

Ans. $b^2 + a^2 = (\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2$
 $= \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cdot \cos \beta + \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \cdot \sin \beta$
 $= 1 + 1 + 2 (\cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta)$
 $= 2 + 2 \cos (\alpha - \beta) \quad (1)$

$$\begin{aligned} b^2 - a^2 &= (\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 \\ &= (\cos^2 \alpha - \sin^2 \beta) + (\cos^2 \beta - \sin^2 \alpha) + 2 \cos (\alpha + \beta) \\ &= \cos (\alpha + \beta) \cos (\alpha - \beta) + \cos (\beta + \alpha) \cos (\alpha - \beta) + 2 \cos (\alpha + \beta) \\ &= 2 \cos (\alpha + \beta) \cdot \cos (\alpha - \beta) + 2 \cos (\alpha + \beta) \end{aligned}$$

$$= \cos (\alpha + \beta) [2 \cos (\alpha - \beta) + 2]$$

$$= \cos (\alpha + \beta) \cdot (b^2 + a^2) \text{ [from (1)]}$$

$$\frac{b^2 - a^2}{b^2 + a^2} = \cos (\alpha + \beta)$$

4. Prove that $\cos \alpha + \cos \beta + \cos \gamma + \cos (\alpha + \beta + \gamma)$

$$= 4 \cos \left(\frac{\alpha + \beta}{2} \right) \cdot \cos \left(\frac{\beta + \gamma}{2} \right) \cdot \cos \left(\frac{\gamma + \alpha}{2} \right)$$

Ans. L. H. S.

$$= \cos \alpha + \cos \beta + \cos \gamma + \cos (\alpha + \beta + \gamma)$$

$$= \cos \alpha + \cos \beta + [\cos \gamma + \cos (\alpha + \beta + \gamma)]$$

$$= 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cdot \cos \left(\frac{\alpha - \beta}{2} \right) + 2 \cos \left(\frac{\alpha + \beta + \gamma + \gamma}{2} \right) \cdot \cos \left(\frac{\alpha + \beta + \gamma - \gamma}{2} \right)$$

$$= 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cdot \cos \left(\frac{\alpha - \beta}{2} \right) + 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cdot \cos \left(\frac{\alpha + \beta + 2\gamma}{2} \right)$$

$$= 2 \cos \left(\frac{\alpha + \beta}{2} \right) \left[\cos \left(\frac{\alpha - \beta}{2} \right) + \cos \left(\frac{\alpha + \beta + 2\gamma}{2} \right) \right]$$

$$= 2 \cos \left(\frac{\alpha + \beta}{2} \right) \left[2 \cos \left(\frac{\frac{\alpha - \beta}{2} + \frac{\alpha + \beta + 2\gamma}{2}}{2} \right) \cdot \cos \left(\frac{\frac{\alpha + \beta + 2\gamma}{2} - \frac{\alpha - \beta}{2}}{2} \right) \right]$$

$$= 2 \cos \left(\frac{\alpha + \beta}{2} \right) \left[2 \cos \left(\frac{\alpha + \gamma}{2} \right) \cdot \cos \left(\frac{\beta + \gamma}{2} \right) \right]$$

$$= 4 \cos \left(\frac{\alpha + \beta}{2} \right) \cdot \cos \left(\frac{\beta + \gamma}{2} \right) \cdot \cos \left(\frac{\gamma + \alpha}{2} \right)$$

5. Prove that $\sin 3x + \sin 2x - \sin x = 4 \sin x \cdot \cos \frac{x}{2} \cdot \cos \frac{3x}{2}$

Ans.

$$\begin{aligned} & (\sin 3x - \sin x) + \sin 2x \\ &= 2 \cos \left(\frac{3x+x}{2} \right) \cdot \sin \left(\frac{3x-x}{2} \right) + \sin 2x \\ &= 2 \cos 2x \cdot \sin x + \sin 2x \\ &= 2 \cos 2x \cdot \sin x + 2 \sin x \cos x \\ &= 2 \sin x [\cos 2x + \cos x] \\ &= 2 \sin x \left[2 \cos x \frac{3x}{2} \cdot \cos \frac{x}{2} \right] \\ &= 4 \sin x \cos \frac{3x}{2} \cos \frac{x}{2} \end{aligned}$$

6. Prove that $2 \cos \frac{\pi}{13} \cdot \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0$

Ans. L.H.S.

$$\begin{aligned} &= 2 \cos \frac{\pi}{13} \cdot \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} \\ &= \cos \left(\frac{\pi}{13} + \frac{9\pi}{13} \right) + \cos \left(\frac{\pi}{13} - \frac{9\pi}{13} \right) + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} \\ &= \cos \frac{10\pi}{13} + \cos \frac{8\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} \end{aligned}$$

$$\begin{aligned}
&= \cos\left(\pi - \frac{3\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} \\
&= \cancel{-\cos\frac{3\pi}{13}} - \cancel{\cos\frac{5\pi}{13}} + \cancel{\cos\frac{3\pi}{13}} + \cancel{\cos\frac{5\pi}{13}} \\
&= 0
\end{aligned}$$

7. Find the value of $\tan(\alpha + \beta)$ Given that

$$\text{Cot } \alpha = \frac{1}{2}, \alpha \in \left(\pi, \frac{3\pi}{2}\right) \text{ and } \text{Sec } \beta = -\frac{5}{3}, \beta \in \left(\frac{\pi}{2}, \pi\right)$$

Ans.

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} \quad (1)$$

$$\text{Cot } \alpha = \frac{1}{2},$$

$$\Rightarrow \tan \alpha = 2$$

$$1 + \tan^2 \beta = \text{Sec}^2 \beta$$

$$1 + \tan^2 \beta = \left(\frac{-5}{3}\right)^2 \left[\because \text{Sec } \beta = \frac{-5}{3}\right]$$

$$\tan \beta = \pm \frac{4}{3}$$

$$\tan \beta = -\frac{4}{3} \left[\because \beta \in \left(\frac{\pi}{2}, \pi\right)\right]$$

put $\tan \alpha$, and $\tan \beta$ in eq. (

$$\tan(\alpha + \beta) = \frac{2 - \frac{4}{3}}{1 - 2\left(-\frac{4}{3}\right)}$$

$$= \frac{2}{11}$$

8. Prove that $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$

Ans. L. H. S =

$$\frac{\frac{1}{\cos 8A} - 1}{\frac{1}{\cos 4A} - 1}$$

$$= \frac{1 - \cos 8A}{1 - \cos 4A} \times \frac{\cos 4A}{\cos 8A}$$

$$= \frac{2 \sin^2 4A}{2 \sin^2 2A} \cdot \frac{\cos 4A}{\cos 8A}$$

$$= \frac{(2 \sin 4A \cdot \cos 4A) \cdot \sin 4A}{2 \sin^2 2A \cdot \cos 8A}$$

$$= \frac{\sin 8A \cdot (\cancel{2 \sin 2A} \cdot \cos 2A)}{\cancel{2 \sin^2 2A} \cdot \cos 8A}$$

$$= \frac{\sin 8A \cos 2A}{\sin 2A \cos 8A}$$

$$= \frac{\tan 8A}{\tan 2A}$$

9. Prove that $\cos^2 x + \cos^2 \left(x + \frac{\pi}{3}\right) + \cos^2 \left(x - \frac{\pi}{3}\right) = \frac{3}{2}$

$$\begin{aligned}
\text{Ans.L. H. S} &= \frac{1 + \cos 2x}{2} + \frac{1 + \cos \left(2x + \frac{2\pi}{3} \right)}{2} + \frac{1 + \cos \left(2x - \frac{2\pi}{3} \right)}{2} \\
&= \frac{1}{2} \left[1 + 1 + 1 + \cos 2x + \cos \left(2x + \frac{2\pi}{3} \right) + \cos \left(2x - \frac{2\pi}{3} \right) \right] \\
&= \frac{1}{2} \left[3 + \cos 2x + \cos \left(2x + \frac{2\pi}{3} \right) + \cos \left(2x - \frac{2\pi}{3} \right) \right] \\
&= \frac{1}{2} \left[3 + \cos 2x + 2 \cos \left(\frac{2x + \frac{2\pi}{3} + 2x - \frac{2\pi}{3}}{2} \right) \cdot \cos \left(\frac{\frac{2\pi}{3} + \frac{2\pi}{3}}{3} \right) \right] \\
&= \frac{1}{2} \left[3 + \cos 2x + 2 \cos 2x \cdot \cos \frac{2\pi}{3} \right] \\
&= \frac{1}{2} \left[3 + \cos 2x + 2 \cos 2x \cdot \cos \frac{2\pi}{3} \right] \\
&= \frac{1}{2} \left[3 + \cos 2x + 2 \cos 2x \cdot \cos \left(\pi - \frac{\pi}{3} \right) \right] \\
&= \frac{1}{2} \left[3 + \cos 2x + 2 \cos 2x \cdot \left(-\frac{1}{2} \right) \right] \\
&= \frac{3}{2}
\end{aligned}$$

10.. Prove that $\cos 2x \cdot \cos \frac{x}{2} - \cos 3x \cdot \cos \frac{9x}{2} = \sin 5x \sin \frac{5x}{2}$

$$\begin{aligned}
\text{Ans.L. H. S} &= \frac{1}{2} \left[2 \cos 2x \cos \frac{x}{2} - 2 \cos 3x \cos \frac{9x}{2} \right] \\
&= \frac{1}{2} \left[\cos \left(2x + \frac{x}{2} \right) + \cos \left(2x - \frac{x}{2} \right) - \cos \left(\frac{9x}{2} + 3x \right) - \cos \left(\frac{9x}{2} - 3x \right) \right]
\end{aligned}$$

$$= \frac{1}{2} \left[\cos \frac{5x}{2} + \cancel{\cos \frac{3x}{2}} - \cos \frac{15x}{2} - \cancel{\cos \frac{3x}{2}} \right]$$

$$= \frac{1}{2} \left[\cos \frac{5x}{2} - \cos \frac{15x}{2} \right]$$

$$= \frac{1}{2} \left[-2 \sin \left(\frac{\frac{5x}{2} + \frac{15x}{2}}{2} \right) \sin \left(\frac{\frac{5x}{2} - \frac{15x}{2}}{2} \right) \right]$$

$$= -\sin 5x \cdot \sin \left(\frac{-5x}{2} \right)$$

$$= \sin 5x \cdot \sin \frac{5x}{2}$$

11. Prove that $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

Ans. L. H. S = $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ$.

$$= \cos 60^\circ \cdot \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cos 40^\circ (2 \cos 20^\circ \cos 80^\circ)$$

$$= \frac{1}{4} \cos 40^\circ [\cos (80^\circ + 20^\circ) + \cos (80^\circ - 20^\circ)]$$

$$= \frac{1}{4} \cos 40^\circ [\cos 100^\circ + \cos 60^\circ]$$

$$= \frac{1}{4} \cos 40^\circ \left[\cos 100^\circ + \frac{1}{2} \right]$$

$$\begin{aligned}
&= \frac{1}{8} (2 \cos 100^\circ \cdot \cos 40^\circ) + \frac{1}{8} \cos 40^\circ \\
&= \frac{1}{8} [\cos (100 + 40^\circ) + \cos (100 - 40^\circ)] + \frac{1}{8} \cos 40^\circ \\
&= \frac{1}{8} [\cos 140^\circ + \cos 60^\circ] + \frac{1}{8} \cos 40^\circ \\
&= \frac{1}{8} \left[\cos 140 + \frac{1}{2} \right] + \frac{1}{8} \cos 40^\circ \\
&= \frac{1}{8} \cos 140^\circ + \frac{1}{16} + \frac{1}{8} \cos 40^\circ \\
&= \frac{1}{8} \cos (180 - 40) + \frac{1}{16} + \frac{1}{8} \cos 40^\circ \\
&= -\frac{1}{8} \cos 40^\circ + \frac{1}{16} + \frac{1}{8} \cos 40^\circ \\
&= \frac{1}{16}
\end{aligned}$$

12. If $\tan x = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$, Find the value of $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$.

Ans. $\pi < x < \frac{3\pi}{2}$ [Given

$\cos x$ is - tive

$$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

$\sin \frac{x}{2}$ is + tive and $\cos \frac{x}{2}$ is - tive.

$$1 + \tan^2 x = \sec^2 x \quad \frac{5}{4}$$

$$1 + \left(\frac{3}{4}\right)^2 = \sec^2 x$$

$$\sec x = \pm$$

$$\cos x = \pm \frac{5}{4}$$

$$\cos x = -\frac{5}{4} \left[\because \pi < x < \frac{3\pi}{2} \right]$$

$$\begin{aligned} \sin \frac{x}{2} &= \sqrt{\frac{1 - \cos x}{2}} \\ &= \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} \end{aligned}$$

$$\begin{aligned} \cos \frac{x}{2} &= -\sqrt{\frac{1 + \cos x}{2}} \\ &= -\sqrt{\frac{1 - \frac{4}{5}}{2}} = -\sqrt{\frac{1}{10}} = \frac{-1}{\sqrt{10}} \end{aligned}$$

$$\tan \frac{x}{2} = \frac{\frac{3}{\sqrt{10}}}{\frac{-1}{\sqrt{10}}} = -3$$